Assessing Measurement System Variation

Example 1: Fuel Injector Nozzle Diameters

Problem

A manufacturer of fuel injector nozzles installs a new digital measuring system. Investigators want to determine how well the new system measures the nozzles.

Data collection

Technicians randomly sample, across all major sources of process variation (machine, time, shift, job change), 9 nozzles that represent those that are typically produced. They code the nozzles to identify the measurements taken on each nozzle.

The first operator measures the 9 nozzles in random order. Then, the second operator measures the 9 nozzles in a different random order. Each operator repeats the process twice, for a total of 36 measurements.

Note For valid measurement system analyses, you must randomly sample and measure parts.

The specification for the nozzle diameters is 9012 ± 4 microns. The tolerance is 8 microns.

Tools

- **Gage R&R Study (Crossed)**

Data set

Nozzle.MPJ

Variable	Description
Nozzle	Fuel injector nozzle measured
Operator	Operator who measured
Run Order	Original run order of the experiment
Diam	Measured diameter of nozzle (microns)

Measurement systems analysis

What is measurement systems analysis

Measurement systems analysis assesses the adequacy of a measurement system for a given application. When measuring the output from a process, consider two sources of variation:

- Part-to-part variation
- Measurement system variation

If measurement system variation is large compared to part-to-part variation, the measurements may not provide useful information.

When to use measurement systems analysis

Before you collect data from your process (for example, to analyze process control or capability), use measurement system analysis to confirm that the measurement system measures consistently and accurately, and adequately discriminates between parts.

Why use measurement systems analysis

Measurement systems analysis answers questions such as:

- Can the measurement system adequately discriminate between different parts?
- Is the measurement system stable over time?
- Is the measurement system accurate throughout the range of parts?

For example:

- Can a viscometer adequately discriminate between the viscosity of several paint samples?
- Does a scale need to be periodically recalibrated to accurately measure the fill weight of bags of potato chips?
- Does a thermometer accurately measure the temperature for all heat settings that are used in the process?

Gage R&R study (crossed)

What is a gage R&R study (crossed)

A crossed gage R&R study estimates how much total process variation is caused by the measurement system. Total process variation consists of part-to-part variation plus measurement system variation. Measurement system variation consists of:

- Repeatability—variation due to the measuring device, or the variation observed when the same operator measures the same part repeatedly with the same device
- Reproducibility—variation due to the measuring system, or the variation observed when different operators measure the same part using the same device

When you estimate repeatability, each operator measures each part at least twice. When you estimate reproducibility, at least two operators must measure the parts. Operators measure the parts in random order, and the selected parts represent the possible range of measurements.

When to use a gage R&R study (crossed)

- Use gage R&R to evaluate a measurement system before using it to monitor or improve a process.
- Use the crossed analysis when each operator measures each part (or batch, for a destructive test) multiple times.

Why use a gage R&R study (crossed)

This study compares measurement system variation to total process variation or tolerance. If the measurement system variation is large in proportion to total variation, the system may not adequately distinguish between parts.

A crossed gage R&R study can answer questions such as:

- Is the variability of a measurement system small compared with the manufacturing process variability?
- Is the variability of a measurement system small compared with the process specification limits?
- How much variability in a measurement system is caused by differences between operators?
- Is a measurement system capable of discriminating between parts?

For example:

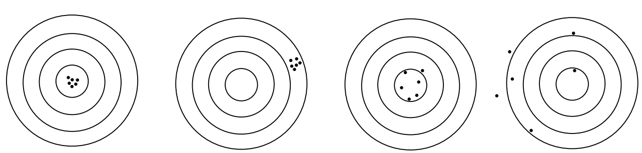
- How much of the variability in the measured diameter of a bearing is caused by the caliper?
- How much of the variability in the measured diameter of a bearing is caused by the operator?
- Can the measurement system discriminate between bearings of different size?

Measurement system error

Measurement system errors can be classified into two categories:

- Accuracy—the difference between the part's measured and actual value
- Precision—the variation when the same part is measured repeatedly with the same device

Errors of one or both of these categories may occur within any measurement system. For example, a device may measure parts precisely (little variation in the measurements) but not accurately. Or a device may be accurate (the average of the measurements is very close to the master value), but not precise (the measurements have large variance). Or a device may be neither accurate nor precise.



accurate and precise | inaccurate but precise | accurate but imprecise | inaccurate and imprecise

Accuracy

The accuracy of a measurement system has three components:

- Bias—a measure of the inaccuracy in the measurement system; the difference between the observed average measurement and a master value
- Linearity—a measure of how the size of the part affects the bias of the measurement system; the difference in the observed bias values through the expected range of measurements
- Stability—a measure of how well the system performs over time; the total variation obtained with a particular device, on the same part, when measuring a single characteristic over time

Precision

Precision, or measurement variation, has two components:

- Repeatability—variation due to the measuring device, or the variation observed when the same operator measures the same part repeatedly with the same device
- Reproducibility—variation due to the measuring system, or the variation observed when different operators measure the same part using the same device

Assessing the measurement system

Use a Gage R&R study (crossed) to assess:

- How well the measuring system can distinguish between parts
- Whether the operators measure consistently

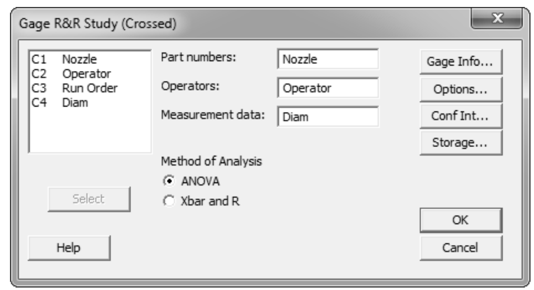
Tolerance

The specification limits for the nozzle diameters are 9012 ± 4 microns. In other words, the nozzle diameter is allowed to vary by as much as 4 microns in either direction. The tolerance is the difference between the specification limits: 9016 – 9008 = 8 microns.

By entering a value in **Process tolerance**, you can estimate what proportion of the tolerance is taken up by the variation in the measurement system.

Gage R&R Study (Crossed)

1. Open Nozzle.MPJ.
2. Choose **Stat** > **Quality Tools** > **Gage Study** > **Gage R&R Study (Crossed)**.
3. Complete the dialog box as shown below.



4. Click **Options**.
5. Under **Process tolerance**, choose **Upper spec - Lower spec** and type *8*.
6. Check **Draw graphs on separate graphs, one graph per page**.
7. Click **OK** in each dialog box.

Interpreting your results

Analysis of variance tables

Minitab uses the analysis of variance (ANOVA) procedure to calculate variance components, and then uses those components to estimate the percent variation due to the measuring system. The percent variation appears in the gage R&R table.

The two-way ANOVA table includes terms for the part (Nozzle), operator (Operator), and operator-by-part interaction (Nozzle*Operator).

If the p-value for the operator-by-part interaction is ≥ 0.05, Minitab generates a second ANOVA table that omits the interaction term from the model. To alter the default Type I error rate of 0.05, click **Options** in the main dialog box. In **Alpha to remove interaction term**, type a new value (for example, 0.3).

Here, the p-value for Nozzle*Operator is 0.707. Therefore, Minitab removes the interaction term from the model and generates a second ANOVA table.

Gage R&R Study - ANOVA Method

Two-Way ANOVA Table With Interaction

Source	DF	SS	MS	F	P
Nozzle	8	46.1489	5.76861	769.148	0.000
Operator	1	0.0400	0.04000	5.333	0.050
Nozzle * Operator	8	0.0600	0.00750	0.675	0.707
Repeatability	18	0.2000	0.01111		
Total	35	46.4489			

α to remove interaction term = 0.05

Two-Way ANOVA Table Without Interaction

Source	DF	SS	MS	F	P
Nozzle	8	46.1489	5.76861	576.861	0.000
Operator	1	0.0400	0.04000	4.000	0.056
Repeatability	26	0.2600	0.01000		
Total	35	46.4489			

Interpreting your results

Variance components

Minitab also calculates a column of variance components (VarComp) and uses the values to calculate %Contribution with the ANOVA method.

The variance components table breaks down the sources of total variability:

- **Total Gage R&R** consists of:
  - **Repeatability**—the variability from repeated measurements by the same operator.
  - **Reproducibility**— the variability when the same part is measured by different operators. (This can be further divided into operator and operator-by-part components.)
- **Part-To-Part**—the variability in measurements across different parts.

Why use variance components?

Use variance components to assess the amount of variation that each source of measurement error and the part-to-part differences contribute to the total variation.

Ideally, differences between parts should account for most of the variability; variability from repeatability and reproducibility should be very small.

Gage R&R

Variance Components

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	0.01167	0.80
Repeatability	0.01000	0.69
Reproducibility	0.00167	0.11
Operator	0.00167	0.11
Part-To-Part	1.43965	99.20
Total Variation	1.45132	100.00

Interpreting your results

Percent contribution

%Contribution is based on the estimates of the variance components. Each value in VarComp is divided by the Total Variation, and then multiplied by 100.

For example, to calculate the %Contribution for Part-to-Part, divide the VarComp for Part-to-Part by the Total Variation and multiply by 100:

(1.43965/1.45132) * 100 ≈ 99.20

Therefore, 99.2% of the total variation in the measurements is due to the differences between parts. This high %Contribution is considered very good. When %Contribution for Part-to-Part is high, the system can distinguish between parts.

Using variance versus standard deviation

Because %Contribution is based on the total variance, the column of values adds up to 100%.

Minitab also displays columns with percentages based on the standard deviation of each term. These columns, labeled %StudyVar and %Tolerance, typically do not add up to 100%.

Because the standard deviation uses the same units as the part measurements and the tolerance, it allows for meaningful comparisons.

Gage R&R

Variance Components

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	0.01167	0.80
Repeatability	0.01000	0.69
Reproducibility	0.00167	0.11
Operator	0.00167	0.11
Part-To-Part	1.43965	99.20
Total Variation	1.45132	100.00

Process tolerance = 8

Gage Evaluation

Source	StdDev (SD)	Study Var (6 × SD)	%Study Var (%SV)	%Tolerance (SV/Toler)
Total Gage R&R	0.10801	0.64807	8.97	8.10
Repeatability	0.10000	0.60000	8.30	7.50
Reproducibility	0.04082	0.24495	3.39	3.06
Operator	0.04082	0.24495	3.39	3.06
Part-To-Part	1.19986	7.19913	99.60	89.99
Total Variation	1.20471	7.22824	100.00	90.35

Number of Distinct Categories = 15

Interpreting your results

Percent study variation

Use %StudyVar to compare the measurement system variation to the total variation.

Minitab calculates %StudyVar by dividing each value in StudyVar by Total Variation and then multiplying by 100.

%StudyVar for gage R&R is

(0.64807/7.22824) * 100 ≈ 8.97%.

Minitab calculates StudyVar as 6 times the standard deviation for each source.

6s process variation

Typically, process variation is defined as 6s, where s is the standard deviation, as an estimate of σ. When data are normally distributed, approximately 99.73% of the data fall within 6 standard deviations (± 3 standard deviations from the mean), and approximately 99% of the data fall within 5.15 standard deviations (± 2.575 standard deviations from the mean).

Note The Automotive Industry Action Group (AIAG) recommends the use of 6s in gage R&R studies.

Gage Evaluation

Source	StdDev (SD)	Study Var (6 × SD)	%Study Var (%SV)	%Tolerance (SV/Toler)
Total Gage R&R	0.10801	0.64807	8.97	8.10
Repeatability	0.10000	0.60000	8.30	7.50
Reproducibility	0.04082	0.24495	3.39	3.06
Operator	0.04082	0.24495	3.39	3.06
Part-To-Part	1.19986	7.19913	99.60	89.99
Total Variation	1.20471	7.22824	100.00	90.35

Number of Distinct Categories = 15

Interpreting your results

Percent tolerance

Comparing the measurement system variation with the tolerance is often informative.

If you enter the tolerance, Minitab calculates %Tolerance, which compares measurement system variation to specifications. %Tolerance is the percentage of the tolerance taken up by the measurement system variability.

Minitab divides the measurement system variation (6*SD for Total Gage R&R) by the tolerance. Minitab multiplies the resulting proportion by 100 and reports it as %Tolerance.

%Tolerance for gage R&R is: (0.64807/8) * 100 ≈ 8.10%

Which metric to use

Use %Tolerance or %StudyVar to evaluate the measuring system, depending on the measuring system.

- If the measurement system is used for process improvement (reducing part-to-part variation), %StudyVar is a better estimate of measurement precision.
- If the measurement system evaluates parts relative to specifications, %Tolerance is a more appropriate metric.

Gage Evaluation

Source	StdDev (SD)	Study Var (6 × SD)	%Study Var (%SV)	%Tolerance (SV/Toler)
Total Gage R&R	0.10801	0.64807	8.97	8.10
Repeatability	0.10000	0.60000	8.30	7.50
Reproducibility	0.04082	0.24495	3.39	3.06
Operator	0.04082	0.24495	3.39	3.06
Part-To-Part	1.19986	7.19913	99.60	89.99
Total Variation	1.20471	7.22824	100.00	90.35

Number of Distinct Categories = 15

Interpreting your results

Total Gage R&R

The %Study Var results indicate that the measurement system accounts for less than 10% of the overall variation in this study. The %Tolerance results indicate that the measurement system variation is less than 10% of the tolerance width.

Total Gage R&R:

- %Study Var—8.97
- %Tolerance—8.10

Remember that Minitab uses different divisors to calculate %Tolerance and %Study Var. Because the range for tolerance (8) is greater than the total study variation (7.22824) in this example, the %Tolerance is lower.

Gage Evaluation

Source	StdDev (SD)	Study Var (6 × SD)	%Study Var (%SV)	%Tolerance (SV/Toler)
Total Gage R&R	0.10801	0.64807	8.97	8.10
Repeatability	0.10000	0.60000	8.30	7.50
Reproducibility	0.04082	0.24495	3.39	3.06
Operator	0.04082	0.24495	3.39	3.06
Part-To-Part	1.19986	7.19913	99.60	89.99
Total Variation	1.20471	7.22824	100.00	90.35

Number of Distinct Categories = 15

Interpreting your results

Number of distinct categories

The Number of Distinct Categories value estimates how many separate groups of parts the system can distinguish.

Minitab calculates the number of distinct categories that can be reliably observed by:

$$\frac{S_{\text{part}}}{S_{\text{measuring system}}} \times \sqrt{2}$$

Minitab truncates this value to the integer except when the value calculated is less than 1. In that case, Minitab sets the number of distinct categories equal to 1.

Number of categories	Means...
< 2	The system cannot discriminate between parts.
= 2	Parts can be divided into high and low groups, as in attributes data.
≥ 5	The system is acceptable (according to the AIAG) and can distinguish between parts.

Here, the number of distinct categories is 15, which indicates the system can distinguish between parts extremely well.

Note The AIAG recommends that the number of distinct categories be 5 or more. See [1] in the reference list.

Gage R&R

Variance Components

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	0.01167	0.80
Repeatability	0.01000	0.69
Reproducibility	0.00167	0.11
Operator	0.00167	0.11
Part-To-Part	1.43965	99.20
Total Variation	1.45132	100.00

Process tolerance = 8

Gage Evaluation

Source	StdDev (SD)	Study Var (6 × SD)	%Study Var (%SV)	%Tolerance (SV/Toler)
Total Gage R&R	0.10801	0.64807	8.97	8.10
Repeatability	0.10000	0.60000	8.30	7.50
Reproducibility	0.04082	0.24495	3.39	3.06
Operator	0.04082	0.24495	3.39	3.06
Part-To-Part	1.19986	7.19913	99.60	89.99
Total Variation	1.20471	7.22824	100.00	90.35

Number of Distinct Categories = 15

Interpreting your results

Components of variation

The Components of Variation chart graphically represents the gage R&R table in the Session window output.

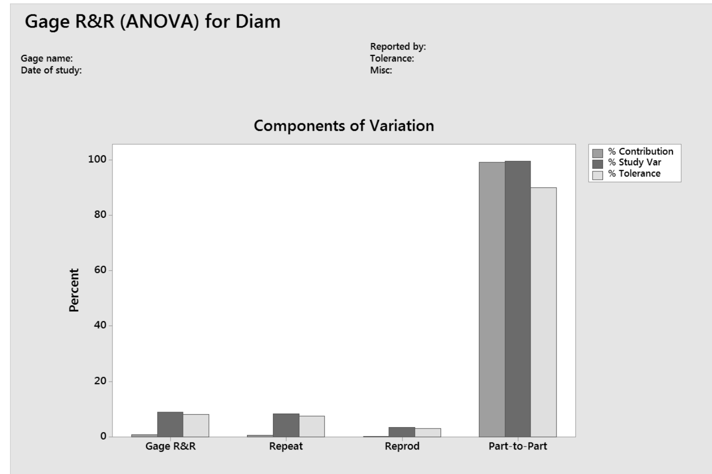
Note In the **Options** subdialog box, you can choose to display these graphs on separate pages.

Each cluster of bars represents a source of variation. By default, each cluster has two bars that correspond to %Contribution and %StudyVar. If you add a tolerance or historical standard deviation, a bar for %Tolerance or %Process appears.

In a good measurement system, the largest component of variation is part-to-part variation. If, instead, large variation is attributed to the measurement system, the measurement system may need correcting.

For the nozzle data, the difference in parts accounts for most of the variation.

Note For the %Study, %Process, and %Tolerance measures, the Repeat and Reprod bars may not add up to the Gage R&R bar because these percentages are based on standard deviations, not on variances.



Interpreting your results

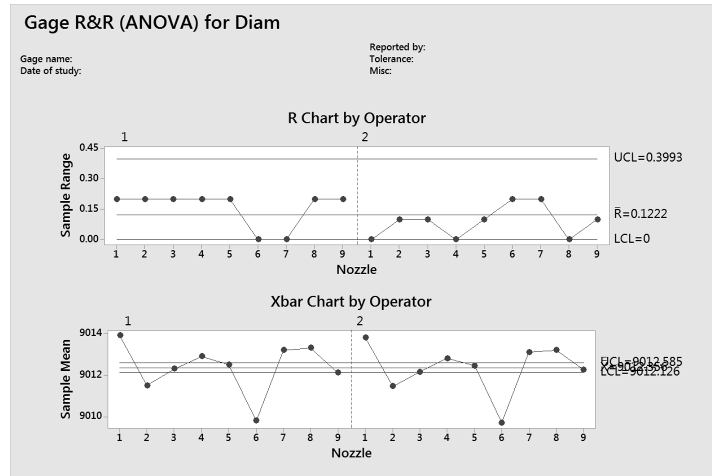
R chart

The R chart is a control chart of ranges that graphically displays operator consistency. An R chart consists of:

- Plotted points, which represent, for each operator, the difference between the largest and smallest measurements of each part. If the measurements are the same, the range = 0. Minitab plots the points by operator so that you can compare the consistency of each operator.
- Center line, which is the grand average of the ranges (the average of all the subgroup ranges).
- Control limits (UCL and LCL) for the subgroup ranges. Minitab uses the within-subgroup variation to calculate these limits.

If any points on the R-chart fall above the upper control limit (UCL), the operator is not consistently measuring the parts. The UCL takes into account the number of times each operator measures a part. If operators measure consistently, the ranges are small relative to the data and the points fall within the control limits.

Note Minitab displays an R chart when the number of replicates is less than 9; otherwise, Minitab displays an S chart.



Interpreting your results

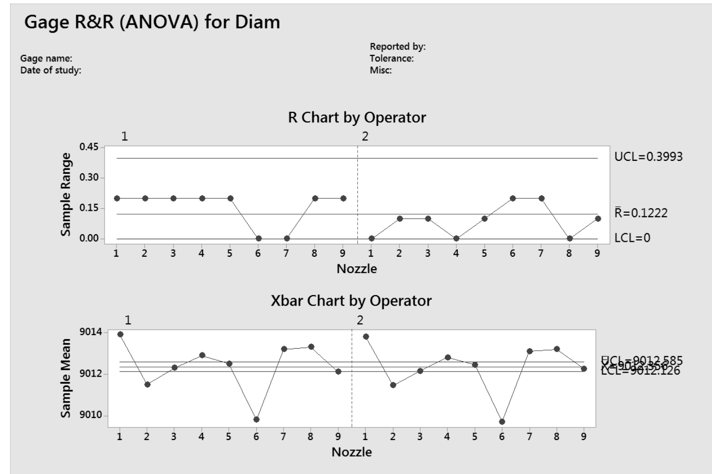
Xbar chart

The Xbar chart compares the part-to-part variation to the repeatability component. The Xbar chart consists of:

- Plotted points, which represent, for each operator, the average measurement of each part.
- Center line, which is the overall average for all part measurements by all operators.
- Control limits (UCL and LCL), which are based on the number of measurements in each average and the repeatability estimate.

Because the parts chosen for a Gage R&R study should represent the entire range of possible parts, this graph ideally shows lack-of-control. It is desirable to observe more variation between part averages than what is expected from repeatability variation alone.

For these data, many points are above or below the control limits. These results indicate that part-to-part variation is much greater than measurement device variation.



Interpreting your results

Operator by part interaction

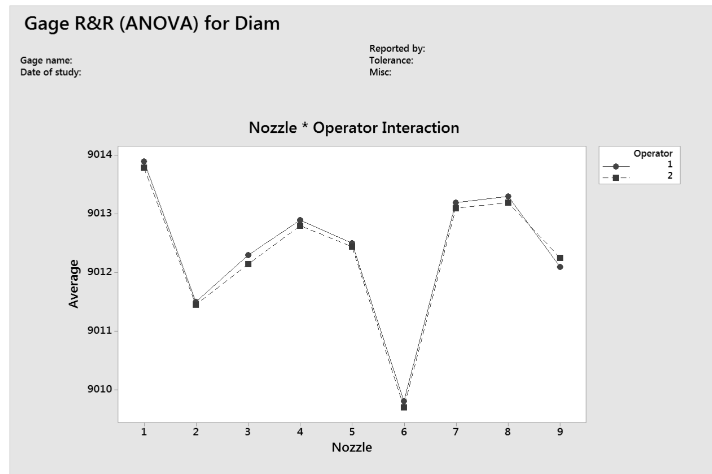
The Nozzle*Operator Interaction plot displays the average measurements by each operator for each part. Each line connects the averages for a single operator.

Ideally, the lines are virtually identical and the part averages vary enough so that differences between parts are clear.

This pattern...	Indicates...
Lines are virtually identical.	Operators are measuring the parts similarly.
One line is consistently higher or lower than the others.	One operator is measuring parts consistently higher or lower than the other operators.
Lines are not parallel, or they cross.	An operator's ability to measure a part depends on which part is being measured (an interaction exists between Operator and Part).

Here, the lines follow one another closely, and the differences between parts are clear. The operators seem to be measuring parts similarly.

Note From the ANOVA table on page 8, the p-value for the interaction is 0.707, which indicates that the interaction is not significant at the α = 0.05 level.



Interpreting your results

Measurements by operator

The By Operator plot can help you to determine whether measurements and variability are consistent across operators.

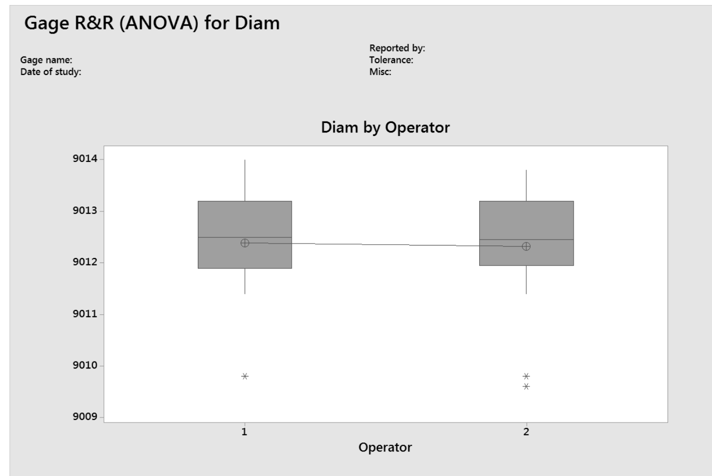
The By Operator graph shows all of the study measurements, arranged by operator. When there are nine or fewer measurements for each operator, dots represent the measurements. When there are more than nine measurements for each operator, Minitab displays a boxplot. For both types of graphs, black circles represent the means, and a line connects them.

If the line is...	Then...
Parallel to the x-axis	The operators are measuring the parts similarly, on average.
Not parallel to the x-axis	The operators are measuring the parts differently, on average.

Also use this graph to assess whether the overall variability in part measurements for each operator is the same:

- Is the spread in the measurements similar?
- Do one operator's measures vary more than the others?

Here, the operators appear to be measuring the parts consistently, with approximately the same variation.



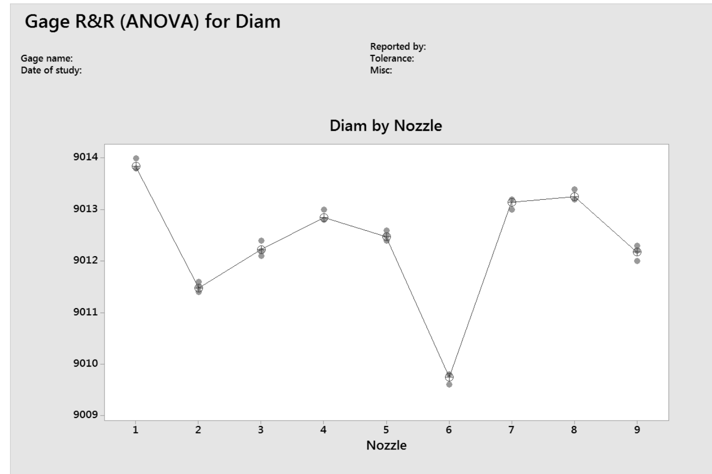
Interpreting your results

Measurements by part

The By Nozzle plot shows all of the measurements in the study arranged by part. Minitab represents the measurements by empty circles and the means by solid circles. The line connects the average measurements for each part.

Ideally:

- Multiple measurements for each part show little variation (the empty circles for each part are close together).
- Averages vary enough so that differences between parts are clear.



Final considerations

Summary and conclusions

The nozzle measuring system contributes very little to the overall variation, as confirmed by both the gage R&R table and graphs.

The variation that is due to the measuring system, either as a percent of study variation or as a percent of tolerance, is less than 10%. According to AIAG guidelines, this system is acceptable.

Additional considerations

Gage R&R (crossed) studies, like other measurement systems analysis (MSA) procedures, are designed experiments. For valid results, randomization and representative sampling are essential.

Final considerations

Additional considerations

Graph patterns that show low measuring-system variation:

Graph	Pattern
R	Small average range
Xbar chart	Narrow control limits and many points out of control
By part	Very similar measurements for each part across all operators, and clear differences between parts
By operator	Straight horizontal line
Operator by part	Overlaid lines

For %Contribution, the AIAG guidelines are:

%Contribution	System is...
1% or less	Acceptable
1% to 9%	Potentially acceptable (depends on the criticality of the measurement, costs, risks, etc.)
9% or greater	Not acceptable

AIAG guidelines for the gage R&R table are:

%Tolerance, %StudyVar %Process	System is...
Under 10%	Acceptable
10% to 30%	Potentially acceptable (depends on the criticality of the measurement, costs, risks, etc.)
Over 30%	Not acceptable

Source: [1] in the reference list.